AN IMPERFECT WELL IN A PIECEWISE RADIALLY INHOMOGENEOUS POROUS STRATUM

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The permeability coefficient k(r) has a discontinuity at $r = r_0$, e.g., as occurs in choking with clay after HCl treatment. A published method [1] is used to solve the problem for given values of the flow in the open part of the well and of the pressure on the supply line.

The isotropic undeformable stratum has a constant thickness H and is filled with a homogeneous incompressible liquid; it is drained by a single central imperfect borehole. The overlying and underlying horizons are impermeable, while the steady-state motion obeys D'Arcy's law. Then the pressure function $\Phi(r, z)$ satisfies (Fig. 1) the equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_i(r)\frac{\partial\Phi^{(i)}}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_i(r)\frac{\partial\Phi^{(i)}}{\partial z}\right)=0, \quad i=1,2$$

$$\Phi^{(i)}=\frac{1}{\mu}(P^{(i)}+\gamma z), \tag{1}$$

and the conditions

$$\frac{\partial\Phi^{(1)}}{\partial r}=\begin{cases}q, & h<z\leqslant H\\ 0, & 0\leqslant z<h\end{cases}, \qquad r=r_1,$$

$$q=\frac{Q\mu}{2\pi r_1 k_1(r_1)(H-h)},$$

$$\frac{\partial\Phi^{(1)}}{\partial z}=\frac{\partial\Phi^{(2)}}{\partial z}=0, \qquad z=0, \qquad z=H, \qquad r_1\leqslant r\leqslant r_2, \tag{2}$$

$$\Phi^{(2)}=\Phi_0=\mathrm{const}, \qquad 0\leqslant z\leqslant H, \quad r=r_2, \tag{3}$$

$$\Phi^{(1)}=\Phi^{(2)}, \qquad k_1\frac{\partial\Phi^{(1)}}{\partial r}=k_2\frac{\partial\Phi^{(2)}}{\partial r},$$

$$0\leqslant z\leqslant H, \qquad r=r_0, \tag{4}$$

where $p^{(i)}$ is the pressure in the region with $k_i(r)$, μ is viscosity, γ is density, Q is flow rate, the well is open between heights h and H, r_1 is the radius of the hole, r_2 is the supply radius, and r and z are cylindrical coordinates.

We further assume that $k(r) \neq 0$ everywhere in the filtration region; then (1) is written as

$$\frac{\partial^2\Phi^{(i)}}{\partial z^2}+\frac{\partial^2\Phi^{(i)}}{\partial r^2}+N_i(r)\frac{\partial\Phi^{(i)}}{\partial r}=0,$$

$$N_i(r)=\frac{d}{dr}\ln(rk_i(r)), \qquad i=1,2. \tag{5}$$

The solution to (5) is expressed via a series [1] as

$$\Phi^{(i)}=c_i+\frac{Q\mu}{2\pi(H-h)}\int_{r_i}^{r}\frac{dr}{rk_i(r)}+\sum_{m=0}^{\infty}F_m^{(i)}(r,z)f_m^{(i)}(r), \tag{6}$$

in which the $F_m^{(i)}(r, z)$ are unknown functions that satisfy

$$\frac{\partial^2 F_m^{(i)}}{\partial z^2}+\frac{\partial^2 F_m^{(i)}}{\partial r^2}=0, \qquad m=0,1,2,\ldots \tag{7}$$

$$C_1=\Phi_0+\frac{Q\mu}{2\pi(H-h)}\left(\int_{r_2}^{r_0}\frac{dr}{rk_2(r)}-\int_{r_1}^{r_0}\frac{dr}{rk_1(r)}\right),$$

$$c_2=\Phi_0. \tag{8}$$

On the $F_m^{(i)}(r, z)$ we impose the conditions

$$F_m^{(i)}(r,z)=\int F_m^{(i)}(r,z)\,dr, \qquad m=1,2,3,\ldots, \tag{9}$$

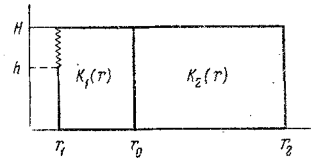

Fig. 1.

and these are applied to $f_0^{(i)}(r)$ to give the recurrence relations

$$\frac{d}{dr}f_0^{(i)}+N_i f_0^{(i)}=0,$$

$$2\frac{d}{dr}f_m^{(i)}+N_i f_m^{(i)}=-\left(\frac{d^2}{dr^2}f_{m-1}^{(i)}+N_i\frac{d}{dr}f_{m-1}^{(i)}\right). \tag{10}$$

In deriving the $f_0^{(i)}(r)$ we consider only particular solutions to (10), i.e., we distinguish a class of solutions to (5) dependent only on the arbitrary functions $F_0^{(i)}(r, z)$. All the other $F_m^{(i)}(r, z)$ are expressed via the previous ones and ultimately in terms of $F_0^{(i)}(r, z)$. From (5) and (10) we get

$$f_0^{(i)}=(rk_i(r))^{-1/2},$$

$$f_m^{(i)}=$$

$$=-\frac{1}{2}(rk_i(r))^{-1/2}\int(rk_i(r))^{-1/2}\left(\frac{d^2}{dr^2}f_{m-1}^{(i)}+N_i\frac{d}{dr}f_{m-1}^{(i)}\right)dr$$

$$(m=1,2,\ldots). \tag{11}$$

The integrals in (11) can be calculated for values of the $k_i(r)$. We take the $F_0^{(i)}(r, z)$ as

$$F_0^{(i)}=\sum_{n=0}^{\infty}(a_n^{(i)}e^{\lambda_n r}+b_n^{(i)}e^{-\lambda_n r})\cos\lambda_n z, \qquad \lambda_n=\frac{n\pi}{H}. \tag{12}$$

From (9) we have for $F_m^{(i)}(r, z)$ that

$$F_m^{(i)}=\sum_{n=0}^{\infty}(a_n^{(i)}e^{\lambda_n r}+(-1)^m b_n^{(i)}e^{-\lambda_n r})\lambda_n^{-m}\cos\lambda_n z,$$

$$m=1,2\ldots. \tag{13}$$

We put

$$\alpha_n^{(i)}(r)=\sum_{n=0}^{\infty}e^{\lambda_n r}\lambda_n^{-m}f_m^{(i)}(r),$$

$$\beta_n^{(i)}(r)=\sum_{n=0}^{\infty}e^{-\lambda_n r}(-1)^m\lambda_n^{-m}f_m^{(i)}(r), \tag{14}$$

and then (6) becomes

$$\Phi^{(i)}=c_i+\frac{Q\mu}{2\pi(H-h)}\int_{r_i}^{r}\frac{dr}{rk_i(r)}+$$

$$+\sum_{n=0}^{\infty}(a_n^{(i)}\alpha_n^{(i)}(r)+b_n^{(i)}\beta_n^{(i)}(r))\cos\lambda_n z. \tag{15}$$

The first boundary conditions of (3) are obeyed, and obedience to the others gives us a linear system of algebraic equations in the $a_n^{(i)}$ and $b_n^{(i)}$

$$a_n^{(1)}(\alpha_n^{(1)})'_{r=r_1}+b_n^{(1)}(\beta_n^{(1)})'_{r=r_2}=-q\sin\lambda_n h/\lambda_n,$$

$$a_n^{(2)}\alpha_n^{(2)}(r_2)+b_n^{(2)}\beta_n^{(2)}(r_2)=0,$$

$$(a_n^{(1)}\alpha_n^{(1)}+b_n^{(1)}\beta_n^{(1)})_{r=r_0}=(a_n^{(2)}\alpha_n^{(2)}+b_n^{(2)}\beta_n^{(2)})_{r=r_0},$$

$$k_1(r_0)(a_n^{(1)}\alpha_n^{(1)}+b_n^{(1)}\beta_n^{(1)})'_{r=r_0}=$$

$$=k_2(r_0)(a_n^{(2)}\alpha_n^{(2)}+b_n^{(2)}\beta_n^{(2)})'_{r=r_0}. \qquad (16)$$

If $h\to 0$, then $q\to Q\mu/2\pi r_1 k_1(r_1)H$ and $\sin\lambda_n h\to 0$, i.e., system (16) becomes homogeneous, and then $a_n^{(i)}=b_n^{(i)}=0$, and the first two terms remain in (6), which give a solution for planar radial filtration to a perfect borehole. If we put $k_1=k_2$, we get the solution for continuously varying $k(r)$. In a similar way we readily get the solution for the case in which the stratum has constant anisotropy and N surfaces of discontinuity in $k(r)$.

REFERENCES

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